NODIVERGENT TURIDAGI O‘ZGARUVCHAN ZICHLIKKA EGA KROSS-DIFFUZIYA SISTEMASINING AVTOMODEL YECHIMI VA SONLI YAQINLASHTIRISH.

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Islomov Bobur Akbar o‘gli

Mirzo Ulug‘bek nomidagi O‘zbekiston Milliy Universiteti talabasi

Annotatsiya: *Ushbu maqolada nodivergent turdagi chiziqsiz parabolik kross-diffuziya tenglamalari sistemasini bir xil bo‘lmagan tarqalish muhitlarida sonli yechishni ko‘rib chiqamiz. Ishning asosiy maqsadi bu boshlang‘ich va chegaraviy shartlarni qanoatlantiradigan taqribiy yechimni qurib olish va uni tenglamani barcha shartlarini qanoatlantirishini taminlashdan iborat. Bu bilan birgalikda sistema uchun sonli yechimlar jadvalini iteratsion jarayonlardan foydalangan holda qurish.*

Kalit so‘zlar*:* *Kross-diffuziya, avtomodel yechimlar, sonli yechimlar, iteratsiya, chiziqlilashtirish, iterativ Picard metodi.*

KIRISH

Dastlab kross-diffuziya jarayonini tasvirlaydigan, ikkinchi tur chegaraviy shartlar asosida berilgan parabolik tipdagi chiziqsiz nodivergent tenglamalar sistemasini matematik modelini ko‘rib chiqaylik:

$$\begin{cases} \rho(x)\dfrac{\partial u}{\partial t} = \dfrac{\partial}{\partial x}\left(v^{m_1-1}\dfrac{\partial u}{\partial x}\right) \\ \rho(x)\dfrac{\partial v}{\partial t} = \dfrac{\partial}{\partial x}\left(u^{m_2-1}\dfrac{\partial v}{\partial x}\right) \end{cases}, \quad (x,t) \in R_+ \times (0,+\infty), \qquad (1)$$

Masala uchun qo‘yilgan boshlang‘ich shartlar:

$$\begin{matrix} u(0,x) = u_0(x) \\ v(0,x) = v_0(x) \end{matrix}, \quad x \in R_+ \qquad (2)$$

Masala uchun qo‘yilgan chegaraviy shartlar:

$$\begin{cases} v^{m_1-1}\left.\dfrac{\partial u}{\partial x}\right|_{x=0} = u^{q_1}(0,t) \\ u^{m_2-1}\left.\dfrac{\partial v}{\partial x}\right|_{x=0} = v^{q_2}(0,t) \end{cases}, \quad t > 0 \qquad (3)$$

Bu yerda, $m_1, m_2 > 1$, $\rho(x) = |x|^n$ -zichlik(diffuziya jarayoni kechayotgan muhit), $u(t,x), v(t,x)$ - biz qidirayotgan tenglamalar sistemasi yechimlari.

O'zaro diffuziya deganda, o'zgaruvchilardan biri bilan tavsiflangan bir ob'ektning fazoviy harakati boshqa o'zgaruvchi tomonidan tasvirlangan boshqa ob'ektning tarqalishi tufayli sodir bo'lishini anglatadi. Bizning muammomizda tizim o'zgaruvchan zichlikdagi ikkita muhitda issiqlik o'tkazuvchanligini tasvirlaydi, oqim uchun chegaraviy muammolar o'rnatiladi.

Muammo issiqlik o'tkazuvchanligi muammolari, filtrlash, o'zaro diffuziya tashish, populyatsiya tizimlarining dinamikasi kabi fizik, kimyoviy va biologik jarayonlarni tavsiflashda muhim rol o'ynaydi. Bundan tashqari ham ko‘plab hayotiy amaliy masalalar ushbu tenglama bilan uzviy bog‘langan.

Bu masalaning o`ziga xosligi shundaki, tenglamalar sistemasi chegaraviy shartlar bilan berilgan. Hozircha faqat Koshi muammosi ko'rib chiqildi.

Ma'lumki, degeneratsiyalangan tenglamalar tizimlari muayyan muhitlarda klassik yechimga ega bo'lmasligi mumkin. Bunda biz fizik ma'noga ega bo'lgan (1) tizimning umumlashtirilgan yechimini o'rganamiz

$$u(x,t),\ \upsilon(x,t)\geq 0,\ \upsilon^{m_1-1}\frac{\partial u}{\partial x},\ u^{m_2-1}\frac{\partial \upsilon}{\partial x}\in C\left(R_+\times(0,+\infty)\right)$$

Albatta. [1,3] shartlarni qanaoatlantiruvchi yechim haqida gap borayapti.

Ko'p sonli ishlar [5-15] raqamli parametrlarning turli qiymatlari uchun (1)-(3) muammoning global echilishi va yechilmasligi shartlarini o'rganishga bag'ishlangan (batafsil ma'lumot uchun foydalanilgan adabiyotlarga qarang). [8, 9] mualliflari o‘rganish vaqtida global echilish va echilmaslik shartlarini o'rganib chiqdilar va mahalliy bo'lmagan diffuziya muammosining portlash vaqtiga yaqin yechimni taqriban topdilar.

$$u_t=u_{xx},\quad \upsilon_t=\upsilon_{xx},\quad x>0,\ 0<T<0, \tag{4}$$

$$-u_x(0,t)=u^{\alpha}\upsilon^{p},\ -\upsilon_x(0,t)=u^{q}\upsilon^{\beta},\ \ 0<t<T, \tag{5}$$

$$u(x,0)=u_0(x),\ \upsilon(x,0)=\upsilon_0(x),\ x>0. \tag{6}$$

Bunda biz $pq\leq(1-\alpha)(1-\beta)$ bo‘lganda (4)-(6) dagi yechimlar global yechim bo‘lishini topamiz.

$$u_t=\left(u^{k_1}\right)_{xx},\quad \upsilon_t=\left(\upsilon^{k_2}\right)_{xx},\quad x\in R_+,\ t>0, \tag{7}$$

$$-\left(u^{k_1}\right)_x(0,t)=\upsilon^{p}(0,t),\quad -\left(\upsilon^{k_2}\right)_x(0,t)=u^{q}(0,t),\quad t>0, \tag{8}$$

$$u(x,0)=u_0(x),\quad \upsilon(x,0)=\upsilon_0(x),\quad x\in R_+, \tag{9}$$

Agar $pq\leq(k_1+1)(k_2+1)/4$ shart bajarilsa (7)-(8) masala yechimlari global bo‘ladi. Boshlang‘ich va chegaraviy shartlar (7)-(9) sistema sonli parametrlari uchun moslab olindi, va ushbu shartlar ostidagi masala yechimi chekli vaqt intervalida portlaydi(cheksiz qiymatlarga erishadi).

$m_i>1\ \ (i=1,2)$ shart ostida (1) tenglamalar tizimi chekli tebranish tarqalish tezligiga ega bo'lgan jarayonlarni tavsiflaydi. $u(x,t),\ \upsilon(x,t)=0$ bilan (1) tenglamalar degenerativdir, shuning uchun (1)-(3) masala degeneratsiya nuqtalarida kerakli silliqlikka ega bo'lmagan umumlashtirilgan yechimni qabul qiladi. Qarayotgan masalamiz uchun umumlashgan yechimlarni quyidagicha quramiz

$$u=(T+t)^{-\alpha_1} f(\xi_1) \qquad v=(T+t)^{-\alpha_2} f(\xi_2)$$
$$\xi_1=\frac{x}{(T+t)^{\gamma_1}} \qquad \xi_2=\frac{x}{(T+t)^{\gamma_2}} \qquad (10)$$
$$\alpha_1,\alpha_2,\gamma_1,\gamma_2$$

Bu yerda $\alpha_1,\alpha_2,\gamma_1,\gamma_2$ nomalum koeffitseyntlar. Biz ushbu (10) taxminiy yechimlarni (1) tenglamalar sistemasiga mos ravishda olib borib qo‘yamiz va soddalashtirishlar amalga oshiramiz. Va quyidagi sistemaga ega bo‘lamiz:

$$\begin{cases}\xi_1^{\,n}(T+t)^{\gamma_1 n}(T+t)^{-\alpha_1-1}(-\alpha_1 f(\xi_1)-\gamma_1\xi_1\frac{\partial f}{\partial\xi_1})=\frac{\partial}{\partial x}\left((T+t)^{-\lambda_2(m_1-1)} f^{m_1-1}(\xi_2)(T+t)^{-\alpha_1-\gamma_1}\frac{\partial f}{\partial\xi_1}\right)\\ \xi_2^{\,n}(T+t)^{\gamma_2 n}(T+t)^{-\alpha_2-1}(-\alpha_2 f(\xi_2)-\gamma_2\xi_2\frac{\partial f}{\partial\xi_2})=\frac{\partial}{\partial x}\left((T+t)^{-\lambda_1(m_2-1)} f^{m_2-1}(\xi_1)(T+t)^{-\alpha_2-\gamma_2}\frac{\partial f}{\partial\xi_2}\right)\end{cases}$$

(11)

Ushbu sistemadagi qidirilayotgan nomalumlarni ayniyatlar usuli yordamida topamiz:

$$\begin{cases}\gamma_1 n-\alpha_1-1=-\alpha_2(m_1-1)-\alpha_1-2\gamma_1\\ \gamma_2 n-\alpha_2-1=-\alpha_1(m_2-1)-\alpha_2-2\gamma_2\end{cases}$$

$$\begin{cases}\gamma_1 n-1=-\alpha_2(m_1-1)-2\gamma_1\\ \gamma_2 n-1=-\alpha_2(m_2-1)-2\gamma_2\end{cases}$$

Sistema yechimlari quyidagi ko‘rinishda bo‘ladi:

$$\alpha_1=\frac{(1-q_2)(2-n)-(m_1-1)(3-n)}{(m_1-1)(m_2-1)(3-n)^2-(1-q_1)(1-q_2)(2-n)^2}$$
$$\alpha_2=\frac{(1-q_1)(2-n)-(m_2-1)(3-n)}{(m_1-1)(m_2-1)(3-n)^2-(1-q_1)(1-q_2)(2-n)^2}$$
$$\gamma_1=\frac{1}{2-n}-\frac{(m_1-1)}{(2-n)}\frac{(1-q_1)(2-n)-(m_2-1)(3-n)}{(m_1-1)(m_2-1)(3-n)^2-(1-q_1)(1-q_2)(2-n)^2} \qquad (12)$$
$$\gamma_2=\frac{1}{2-n}-\frac{(m_2-1)}{(2-n)}\frac{(1-q_2)(2-n)-(m_1-1)(3-n)}{(m_1-1)(m_2-1)(3-n)^2-(1-q_1)(1-q_2)(2-n)^2}$$

Keyingi qadamda ushbu sistema hosil bo‘ladi:

$$\begin{cases}\frac{\partial}{\partial\xi_1}\left(f^{m_1-1}(\xi_2)\frac{\partial f}{\partial\xi_1}\right)+\xi_1^{\,n}(\alpha_1 f(\xi_1)+\gamma_1\xi_1\frac{\partial f}{\partial\xi_1})=0\\ \frac{\partial}{\partial\xi_2}\left(f^{m_2-1}(\xi_1)\frac{\partial f}{\partial\xi_2}\right)+\xi_2^{\,n}(\alpha_2 f(\xi_2)+\gamma_2\xi_2\frac{\partial f}{\partial\xi_2})=0\end{cases} \qquad (13)$$

Nomalum f funksiyani $f=(a-b\xi^{k_1})^{k_2}$ ushbu ko'rinishda qidiramiz, uni (13) sistemaga olib borib qo'yamiz va chegaraviy shartlarni hisobga olgan holda nomalum koeffitseyntlarni topamiz:

$$a=1$$

$$b=\frac{\gamma}{n+2}\left(m_2-1\right)$$

$$k_1=n+2$$

$$k_2=\frac{1}{m_2}-1$$

Bunda $u(t,x),v(t,x)$ yechimlar chegaralangan va $u(t,x)=0$, va albatta x argumentimiz ushbu $x\geq l(t)$ shartni qanoatlantirishi kerak. *l(t)* ning ko'rinishini quyidagicha deb olamiz:

$$l(t)=(T+t)^{\gamma_1}\left(\frac{a}{b}\right)^{\frac{1}{k_1}};$$

SONLI YECHISH

Qaralayotgan soha qismlarga ajratiladi va uni quyidagicha yozamiz:

$$\Omega=\left\{(x_i,t_j):x_i=ih;h=\frac{l(t)}{n};t_j=j\tau;\tau=\frac{T}{m};i=\overline{0,n};\ j=\overline{0,m}\right\}$$

Sistemamizni unga ekvivalent bo'lgan ayirmali sxema bilan almashtiramiz:

$$\begin{cases} x_i^n\dfrac{u_i^j-u_i^{j-1}}{\tau}=\dfrac{1}{h}\left(a1_{i+1}^j(v)\dfrac{u_{i+1}^j-u_i^j}{h}-a1_i^j(v)\dfrac{u_i^j-u_{i-1}^j}{h}\right)\\ x_i^n\dfrac{v_i^j-v_i^{j-1}}{\tau}=\dfrac{1}{h}\left(a2_{i+1}^j(u)\dfrac{v_{i+1}^j-v_i^j}{h}-a2_i^j(u)\dfrac{v_i^j-v_{i-1}^j}{h}\right)\end{cases}$$

Bu yerda $a_i^j(v)=(v_i^j)^{m_1-1}$ Picardning iteratsion metodidan foydalanamiz va sistema ko'rinishi quyidagicha bo'ladi:

$$\begin{cases} x_i^n\dfrac{\overset{s\ j}{u_i}-u_i^{j-1}}{\tau}=\dfrac{1}{h}\left((\overset{s-1j}{v}_{i+1})^{m_1-1}\dfrac{\overset{s\ j}{u}_{i+1}-\overset{s\ j}{u}_i}{h}-(\overset{s-1j}{v}_i)^{m_1-1}\dfrac{\overset{s\ j}{u}_i-\overset{s\ j}{u}_{i-1}}{h}\right)\\ x_i^n\dfrac{\overset{s\ j}{v_i}-v_i^{j-1}}{\tau}=\dfrac{1}{h}\left((\overset{s-1j}{u}_{i+1})^{m_2-1}\dfrac{\overset{s\ j}{v}_{i+1}-\overset{s\ j}{v}_i}{h}-(\overset{s-1j}{u}_i)^{m_2-1}\dfrac{\overset{s\ j}{v}_i-\overset{s\ j}{v}_{i-1}}{h}\right)\end{cases}$$

$s=0,1,2...$ iteratsiya tartibi, chegaraviy shartlargacha davom etadi, bu yerda $\varepsilon>0$;

Ushbu iteratsion sistemani oshkormas haydash usulidan foydalanib yechamiz

$$\begin{cases} A_i^1\overline{u}_{i-1}-C_i^1\overline{u}_i+B_i^1\overline{u}_{i+1}=-F_i^1\\ A_i^1v_{i-1}-C_i^1\overline{v}_i+B_i^1\overline{v}_{i+1}=-F_i^1\end{cases}$$

$$\begin{cases} A_i^1 = \dfrac{\tau}{h^2}\left(v_i^j\right)^{m_1-1} \\ C_i^1 = \dfrac{\tau}{h^2}\left(\left(v_{i+1}^j\right)^{m_1-1} + \left(v_i^j\right)^{m_1-1}\right) + x_i^n \\ B_i^1 = \dfrac{\tau}{h^2}\left(v_{i+1}^j\right)^{m_1-1} \end{cases} \quad \text{va} \quad \begin{cases} A_i^2 = \dfrac{\tau}{h^2}\left(u_i^j\right)^{m_2-1} \\ C_i^2 = \dfrac{\tau}{h^2}\left(\left(u_{i+1}^j\right)^{m_2-1} + \left(u_i^j\right)^{m_2-1}\right) + x_i^n \\ B_i^2 = \dfrac{\tau}{h^2}\left(u_{i+1}^j\right)^{m_2-1} \end{cases}$$

XULOSA

Bir xil bo'lmagan zichlikka ega (1) nodivergent shakldagi kross-diffuziya sistemasi uchun avtomodel tenglamalar tizimi tuziladi va raqamli parametrlar qiymatga qarab ushbu tizimning eng yaxshi avtomodel yechimi topiladi. Topilgan yechim cheksizlikda yo'q bo'ladigan barcha yechimlarning asimptotikasi ekanligi isbotlangan. Bu masalaning o`ziga xosligi shundaki, tenglamalar sistemasi chegaraviy shartlar bilan berilgan. Hozircha faqat Koshi masalasi ko'rib chiqildi. Ko'rib chiqilayotgan masalani sonli yechish uchun dastlabki yaqinlashishlar sifatida yechimlar asimptotikasidan foydalanish mumkin.

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