GEOMETRIYANI CHUQURLASHTIRIB O'RGANISHDA KOORDINATA-VEKTOR USULIDAN FOYDALANISHNING METODIK XUSUSIYATLARI

https://doi.org/10.5281/zenodo.10557766

To'xtasinova Nafisa Imomovna
FarDU Matematika kafedrasi assistenti

Annatatsiya: *Ushbu maqolada geometriya fanini o'rgatishda har bir mavzuga alohida yondashish, mavzular uchun metodlarni tanlashda VR texnologiyalarini keng qo'llash ham ahamiyatli ekanligi ta'kidlab o'tilgan.*

Kalit so'z: *vektor, fazo, munosabat, kesma, geometriya, koordinata-vektor usuli*

Bu yerda tavsiya etiladigan material masalalar yechish bo'yicha fakultativ mashg'ulotlarda (chunki har qanday fakultativ mashg'ulot qiyinlashtirilgan masalalarni yechishni ko'zda tutadi) ko'rilishi, matematikani chuqurlashtirilgan o'rganish sinflari dasturiga kiritilishi mumkin. Bunda quyidagi bo'limlar qaraladi:

a) to'g'ri burchakli koordinatalar sistemasida koordinata-vektor usuli (geometriya darslaridagidan murakkabroq masalalarni yechish).

Masalalarni to'g'ri burchakli koordinatalar sistemasida koordinata-vektor usulida yechish geometriyani chuqurlashtirilgan o'rganishda yechishning murakkabligi bilan farq qiladi. Bu masalalarning murakkabligi vaziyati masala shartida ko'zda tutilmagan nuqtalarning koordinatalarini topish, vektor munosabatni tuzishda boshqa vektor munosabatlarni tuzishga murojaat etish va, shuningdek, qo'shimcha tadqiqotlar o'tkazishdan iborat.

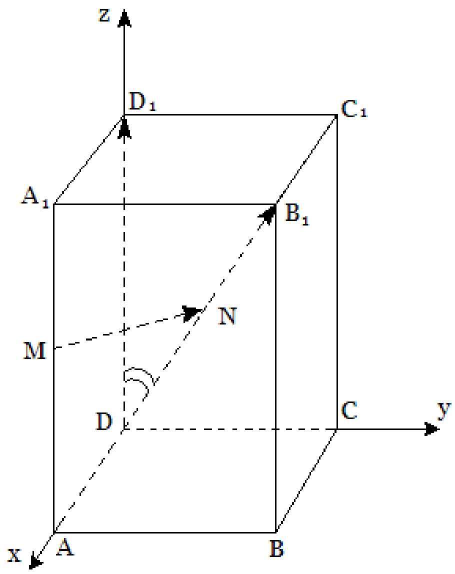
Ushbu masalari ko'raylik.

1-masala. $ABCD\ A1B1C1D1$ to'g'ri burchakli parallelepipedda $AD : DC = a{:}\,b.$ $A1A$ va $DB1$ kesmalarni qanday nisbatda bo'ladi?

Yechilishi. 1. To'g'ri burchakli koordinatalar sistemasini 1-rasmda ko'rsatilganidek kiritamiz. $|DA| = a;\ |DC| = c;\ |DD1| = c$ bo'lsin. Nuqtalarning koordinatalarini aniqlaymiz:

$A(a;0;0), B1(a;b;0), C(0;b;0), A1(a;0;c), D(0;0;0).$

2. MN bu $A1A$ va $DB1$ to‘g‘ri chiziqlarning umumiy perpendikulyari ekanligini hisobga olib, muhokamaga $\overline{MN}$, $\overline{A_1A}$ va $\overline{DB_1}$ vektorlarni kiritamiz.



1-rasm

Bu yerda masala talabiga mos asosiy vektor munosabatni tuzishda murakkablik yuzaga keladi. Mazkur holda izlanayotgan munosabatni $\overline{MN} \perp \overline{A_1A}$ va $\overline{MN} \perp \overline{DB_1}$ shartdan topamiz:

$$\begin{cases} \overline{MN} \cdot \overline{A_1A} = 0 \\ \overline{MN} \cdot \overline{DB_1} = 0 \end{cases}$$

$\overline{MN}$ ni vektorlar yig‘indisi $\overline{MA} + \overline{AD} + \overline{DN}$ ko‘rinishida ifodalab, quyidagiga ega bo‘lamiz:

$$\begin{cases} \left(\overline{MA} + \overline{AD} + \overline{DN}\right) \cdot \overline{A_1A} = 0 \\ \left(\overline{MA} + \overline{AD} + \overline{DN}\right) \cdot \overline{DB_1} = 0 \end{cases}.$$

3. Asosiy vektor munosabatga kiruvchi vektorlarning koordinatalarini kiritamiz. $A, A1, D$ va $V1$ nuqtalarning koordinatalarini bilgan holda $\overline{AA_1}, \overline{DB_1}$ va $\overline{AD}$ vektorlarning koordinatalarini topamiz: $\overline{A_1A}(0;0;-c)$, $\overline{DB_1}(a;b;c)$, $\overline{AD}(-a;0;0)$.

$\overline{MA}$ va $\overline{A_1A}$ vektorlar kollinear bo‘lganligi uchun $\overline{MA} = p \cdot \overline{A_1A}$, $\overline{MA}(0;0;-pc)$. $\overline{DN}$ vektor $\overline{DB_1}$ ga kollinear bo‘lganligini uchun bunday yozamiz:

$$\overline{DN} = q\overline{DB_1}, \overline{DN}(qa;qb;qc).$$

4. Asosiy vektor munosabatni koordinata shaklda yozamiz:

$$\begin{cases}\left[\left(\overline{0;0;-pc}\right)+\left(\overline{a;0;0}\right)+\left(\overline{qa;qb;qc}\right)\right]\cdot\left(\overline{0;0;-c}\right)=0\\ \left[\left(\overline{0;0;-pc}\right)+\left(\overline{-a;0;0}\right)+\left(\overline{qa;qb;qc}\right)\right]\cdot\left(\overline{a;b;c}\right)=0\end{cases}$$

yoki

$$\begin{cases}(qa-a)\cdot 0+(qb)\cdot 0+(qc-pc)\cdot(-c)=0\\ (qa-a)\cdot a+(qb)\cdot b+(qc-pc)\cdot c=0\end{cases}$$

Bu sistemani *r* va *q* ga nisbatan yechib,

$$p=q=\frac{a^2}{a^2+b^2}$$

ni topamiz.

Demak,

$$\frac{MA}{MA_1}=\frac{DN}{NB_1}=\frac{a^2}{a^2+b^2}.$$

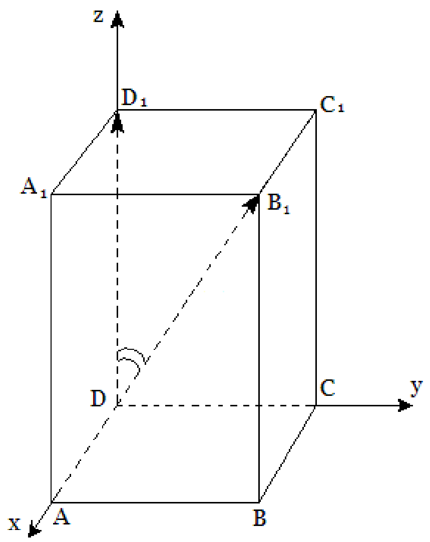
Bu masala biroz murakkabroq. Asosiy qiyinchilik masala shartida vaziyati berilmagan *M* va *N* nuqtalarning koordinatalarini topishdan, va, shuningdek, masala talabini qanoatlantiruvchi asosiy vektor munosabatni tuzishdan iborat.

Navbatdagi masala koordinata-vektor usulida an’anaviy usul bilan birgalikda yechilgan. Masalani bunday “oson” yechish o‘quvchilarga geometrik dalillar va algebraik (koordinataviy) ifodalar orasidagi o‘zaro bog‘lanishini tezroq payqab olish imkonini beradi.

Masalani ko‘raylik.

2-masala. $ABCD\ A1B1C1D1$ to‘g‘ri to‘rtburchakli parallelepipedning DB_1 diagonali *AD* va *DC* qirralar bilan 60°li burchak hosil qiladi. U DD_1 qirra bilan qanday burchak hosil qiladi?

Yechish. To‘g‘ri burchakli koordinatalar sistemasini 28-rasmda ko‘rsatilganidek kiritamiz.



2-rasm

$|DA|=a, |DB|=b, |DD_1|=c$. U holda D, B_1 va D_1 nuqtalarning koordinatalari $D(0;0;0;), B_1(a;b;c), D_1(0;0;c)$ bo‘lgadi.

2. DB_1 diagonal bilan DD_1 qirra orasidagi burchakni aniqlash uchun muhokamaga $\overline{DB_1}$ va $\overline{DD_1}$ vektorlarni kiritamiz va masala talabiga mos vektor munosabatni yozamiz:

$$\cos\varphi = \frac{\overline{DB_1}\cdot\overline{DD_1}}{|\overline{DB_1}|\cdot|\overline{DD_1}|} \qquad (*)$$

3.Bu manosabatga kiruvchi vektorlarning koordinatalarini aniqlaymiz: $\overline{DB_1}(a;b;c)$, $\overline{DD_1}(a;0;c)$. Ularni (*) vektor munosabatga qo‘yamiz:

$$\cos\varphi = \frac{(\overline{a;b;c})\cdot(\overline{0;;c})}{\sqrt{a^2+b^2+c^2}\cdot\sqrt{c^2}} =$$

$$= \frac{c^2}{\sqrt{a^2+b^2+c^2}\cdot c} = \frac{c}{\sqrt{a^2+b^2+c^2}}. \qquad (1)$$

Kiritilgan *a,b* va *s* belgilashlarni masala shartida berilgan ma’lumotlar orqali ifodalaymiz:

$$DA = a = |\overline{DB_1}|\cdot\cos 60^o = \frac{DB_1}{2}; \qquad (2)$$

$$DC = b = |\overline{DB_1}|\cdot\cos 60^o = \frac{DB_1}{2}; \qquad (3)$$

$$DD_1 = c = |\overline{DB_1}|\cdot\cos\varphi. \qquad (4)$$

(1)-(4) tengliklardan quyidagiga ega bo‘lamiz:

$$\cos\varphi = \frac{|\overline{DB_1}|\cdot\cos\varphi}{\sqrt{\frac{1}{2}|\overline{DB_1}|^2 + |\overline{DB_1}|^2\cdot\cos^2\varphi}} = \frac{|\overline{DB_1}|\cdot\cos\varphi}{|\overline{DB_1}|\sqrt{\frac{1}{2}+\cos^2\varphi}}.$$

Bu yerda $1 + \frac{1}{\sqrt{\frac{1}{2}+\cos^2\varphi}}$. Bu tenglikning ikkala qismini kvadratga ko‘tarib va tegishli almashtirishlarni bajarib quyidagini hosil qilamiz:

$$\frac{1}{2}+\cos^2\varphi = 1; \quad \cos^2\varphi = \frac{1}{2}.$$

$\varphi < 90^\circ$ligini hisobga olib yozamiz:

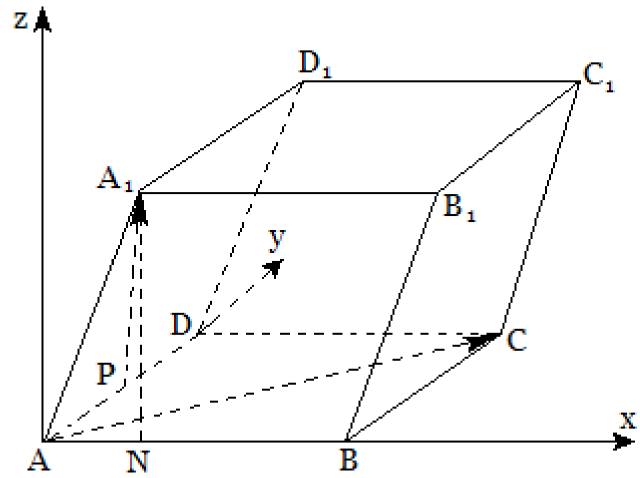
$$\cos\varphi = \frac{\sqrt{2}}{2}, \quad \varphi = \arccos\frac{\sqrt{2}}{2}.$$

4. Shunday qilib, DB_1 diagonal va DB_1 qirra orasidagi burchak $45\,^{\circ}$ga teng.

Endi og‘ma parallelopired xossalarini o‘rganishda koordinata-vektor usuli qo‘llaniladigan masalaga misol keltiramiz.

3-masala. Barcha qirralari o‘zaro teng $ABCD\,A_1B_1C_1D_1$ parallelepiped berilgan. $ABCD$ yo‘q-kvadrat. $\angle A_1AB = \angle A_1AD = \varphi,\ 0 < \varphi < 90\,^{\circ}$. AA_1 qirraning asos tekisligiga og‘ish burchagini toping.

Yechish. 1. To‘g‘ri burchakli koordinatalar sistemasini 3-rasmdagidek kiritamiz.



3-rasm

1. Parallelepiped qirrasini 1 ga teng deb olamiz. Bizga kerakli nuqtalarning koordinatalarini aniqlaymiz: $A(0;0;0)$, $C(1;1;0)$. A_1 nuqtaning koordinatalarini aniqlash uchun quyidagicha yo‘l tutamiz: A_1 nuqtadan mos ravishda *AV* va *AD* tomonlarga *AN* va *AR* perpendikulyarlarni tushiramiz. AA_1N va AA_1R uchburchaklardan *AN* va *AR* ni aniqlaymiz, ular A_1 nuqtaning birinchi va ikkinchi koordinatalari bo‘ladi.

Quyidagi ega bo‘lamiz: $AN = cos\varphi$; $AP = cos\varphi$, ya’ni $A_1(cos\varphi; cos\varphi; z)$.

2. AA_1 qirraning asos tekisligiga og‘malik burchagini aniqlash uchun $\overline{AA_1}$ va $\overline{AC}$ vektorlar orasidagi burchakni aniqlash yetarlidir. Xaqiqatan, A_1 nuqta koordinatalarining ko‘rinishidan kelib chiqadiki, A_1 uchdan *ABCD* tekisligiga tushirilgan perpendikulyar *AS* diagonalga tushadi, ya’ni $\angle A_1AC$ burchak AA_1 qirraning asosga og‘ish burchagidir. $\overline{AA_1}$ va $\overline{AC}$ vektorlani muhokamaga kiritamiz va masala talabiga mos asosiy vektor munosabatni yozamiz:

$$cos\varphi = \frac{\overline{AA_1} \cdot \overline{AC}}{|\overline{AA_1}| \cdot |\overline{AC}|}.$$

3. $\overline{AA_1}$ va $\overline{AC}$ vektorlarning koordinatalarini aniqlaymiz: $\overline{AC}(1;1;0)$, $\overline{AA_1}(\cos\varphi;\cos\varphi;z)$.

Asosiy vektor munosabatni koordinatalarda yozamiz:

$$\cos\alpha = \frac{\overline{(\cos\varphi;\cos\varphi;z)}\cdot\overline{(1;0;0)}}{\sqrt{2}} = \frac{2\cos\varphi}{\sqrt{2}} = \sqrt{2}\cos\varphi .$$

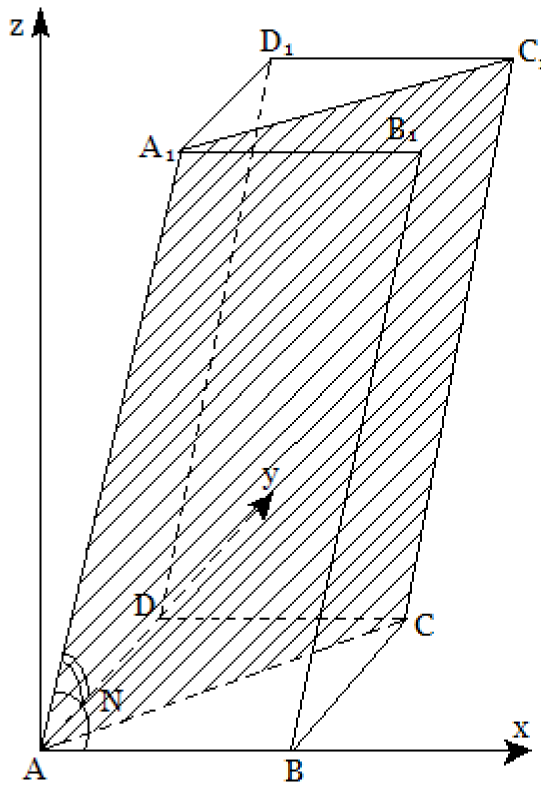
4. Shunday qilib

$$\cos(\angle A_1AC) = \sqrt{2}\cos\varphi, \quad \angle A_1AC = \arccos\left(\sqrt{2}\cos\varphi\right).$$

Navbatdagi masala diagonal kesim yuzini aniqlashga oiddir.

4-masala. *ABCD* $A_1B_1C_1D_1$ parallelepipedning asosi tomoni *a* bo‘lgan kvadrat, uning yon qirrasi *b* ga teng. AA_1 yon qirra asosining tomonlari bilan α ga teng o‘tkir burchak hosil qiladi. Parallelepiped AA_1SS_1 va BB_1D_1D diagonal kesimlarining yuzalarini toping.

Yechish. 1-3. AA_1S_1S va BB_1D_1D diagonal kesimlar yuzlarini ushbu formulalardan aniqlaymiz (4-rasm)



4-rasm

$$S_{AA_1C_1C} = |AC|\cdot|AA_1|\cdot\sin(\angle A_1AC), \quad (1)$$

$$S_{BB_1D_1D} = |BD|\cdot|BB_1|\cdot\sin(\angle B_1BD) \quad (2)$$

masalaning yechilishidan $\cos(\angle A_1AC) = \sqrt{2}\cos\alpha$ ekanligi kelib chiqadi.

4. Endi esa $\sin(\angle A_1AC)$ ni aniqlaymiz:

$$\sin(\angle A_1AC)=\sqrt{1-2\cos^2\alpha},\ |AC|=a\sqrt{2},\ |AA_1|=b.$$

Bu ma’lumotlarni (1) ga qo‘yamiz:

$$S_{A_1C_1CA}=a\sqrt{2}\cdot b\cdot\sqrt{1-2\cos^2\alpha}=ab\sqrt{-2\cos 2\alpha},$$

$$\cos(\angle B_1BD)=\sqrt{2}\cos(\pi-\alpha)=-\sqrt{2}\cos\alpha.$$

Endi $\sin(\angle B_1BD)$ ni aniqlaymiz:

$$\sin(\angle B_1BD)=\sqrt{1-2\cos^2\alpha}=\sqrt{-\cos 2\alpha}.$$

Bu ma’lumotlarni (2) va qo‘yamiz:

$$S_{BB_1D_1B}=a\sqrt{2}\cdot b\cdot\sin(\angle B_1BD)=ab\sqrt{-2\cos 2\alpha}.$$

Masala $45^\circ<\alpha<90^\circ$da ma’noga ega.

FOYDALANILGAN ADABIYOTLAR:

1. Tukhtasinov, D. (2018). Development of logical thinking of pupils of 5-9th grades in the lessons of mathematics. *Zbiór artykułów naukowych recenzowanych*, *209*(22), 586-587.

2. Farhodovich, T. X. D. (2023). Boshlang ‘ich sinf o ‘quvchilarining tafakkurini rivojlantirishning psixologik va pedagogik jihatlari. *Ijtimoiy fanlarda innovasiya onlayn ilmiy jurnali*, *3*(3), 24-28.

3. Tukhtasinov, D. F. (2018). Developing Logical Thinking of 5-9th Year Students at Mathematics Lessons. *Eastern European Scientific Journal*, (2).

4. To'xtasinov, D. F. (2018). Didactic Bases Of Development Of Logical Thinking In Schoolchildren. *Central Asian Journal of Education*, *2*(1), 68-74.

5. To‘xtasinov, D. F., Gafurova, M. A., & qizi Abdullayeva, S. H. (2023). BOSHLANG ‘ICH SINFDA IJODIY TAFAKKURNI RIVOJLANTIRISHNING PEDAGOGIK ASOSLARI. *Educational Research in Universal Sciences*, *2*(4), 729-733.

6. To‘xtasinov, D. F., & qizi Abdupattoyeva, D. A. (2023). BOSHLANG ‘ICH SINF O ‘QUVCHILARINING GEOMETRIK TASAVVURINI SHAKLLANTIRISHDA FOYDALANILADIGAN METODLAR. *Educational Research in Universal Sciences*, *2*(4), 771-773.

7. To‘xtasinov, D. F. qizi Abdullayeva, SH (2023). MATEMATIKA DARSLARIDA IJODIY TAFAKKURNI RIVOJLANTIRISH SHARTLARINING DIDAKTIK KOMPLEKSINI AMALIYOTDA QO‘LLASH YO‘LLARI. *Educational Research in Universal Sciences*, *2*(2), 613-616.

8. Farxodovich, T. D., & Azatovich, M. O. (2023). DISKRET MATEMATIKADA MARSHRUTLAR VA ZANJIRLAR. *PROSPECTS AND MAIN TRENDS IN MODERN SCIENCE*, *1*(5), 67-71.

9. Тўҳтасинов, Д. Ф., & Саидов, М. И. (2023). ИККИНЧИ ТАРТИБЛИ БИР ЖИНСЛИ БЎЛМАГАН БУЗИЛАДИГАН БИР ОДДИЙ ДИФФЕРЕНЦИАЛ ТЕНГЛАМА УЧУН ИККИ НУҚТАЛИ 4-ЧЕГАРАВИЙ МАСАЛАНИ ГРИН ФУНКЦИЯЛАРИ УСУЛИ БИЛАН ЕЧИШ. *PROSPECTS AND MAIN TRENDS IN MODERN SCIENCE*, *1*(5), 72-75.

10. Imomovna, T. N. Akbarova Sevara Husanboy qizi.(2023). *APPLICATION OF INTERACTIVE TEACHING METHODS IN MATHEMATICS LESSONS. Academicia Globe: Inderscience Research*, *4*(01), 86-93.

11. Nishonboyev, A. S., Akbarova, S. X., & Tuhtasinova, N. I. (2023). Teaching Students Independent Thinking in the Process of Teaching Mathematics in the System of Continuing Education. *European journal of innovation in nonformal education*, *3*(3), 9-11.

12. To‘xtasinova, N. I., & Mirzaabdullayev, I. (2023). Qutb koordinatalar sistemasida uchburchak yuzini hisoblashni matematik modeli. *Educational Research in Universal Sciences*, *2*(4), 518-522.

13. To‘xtasinova, N. (2023). GEOMETRIYA DARSLARIDA STEREOMETRIK MASALALARNI YECHISH METODIKASI. *Research and implementation*, *1*(3), 4-10.