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# ANALYSIS OF MATHEMATICAL MODELS OF UNDERGROUND HEAT EXCHANGE DEVICES WITH A SHALL VERTICAL WELL

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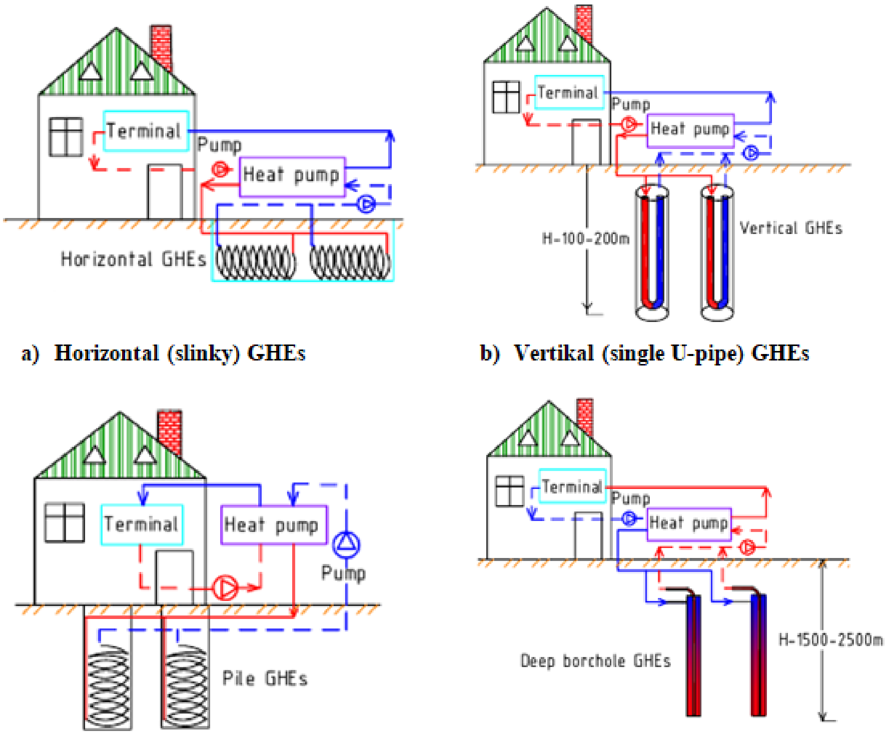
**Abstract**: *Geothermal energy sources are widely used as a renewable energy source in heating and cooling systems of buildings. The thermal characteristics of underground heat exchangers are the most important for efficient operation of geothermal heat pumps (GHP). This article reviews the latest developments in three types of GHPs commonly used in large GHP systems, vertical well GHPs, piled GHPs, and horizontal well GHPs. Also, analyzes of analytical and numerical models proposed for the study of heat transfer processes taking into account different geological conditions are presented.*

**Keywords**: *geothermal heat pumps, vertical well geothermal heat pumps, horizontal well heat pumps, heating and cooling, horizontal underground heat exchangers*

## I.INTRODUCTION

Developing the use of renewable energy resources in heating and cooling systems of buildings, reducing the impact on the environment and the amount of greenhouse gas emissions is one of the urgent tasks. Among the renewable energy sources used, geothermal energy is attracting great interest due to the reliability and ubiquity of the geothermal source. There are six main categories of geothermal energy use: geothermal heat pumps (GHP) for use in bathing and swimming pools, room heating, greenhouse heating, aquaculture pond heating, and industrial use [1]. GHPs are the most widely used in the world, and their total installed capacity is 77,547 MW. This value is 71.6% of the total installed capacity of direct use of geothermal energy in 2020. According to the information of the International Geothermal Congress (IGC), this value increased 1.84 times the annual consumption of geothermal energy compared to 2015. GHPs play an important role in residential and commercial buildings to save energy, reduce CO2 emissions and prevent air pollution [2]. A simple GHP system consists of an underground heat exchanger, a heat pump and indoor heat supply systems. Open use of geothermal energy is usually indirect, where the main device is an underground heat exchanger. There are four types of underground heat exchangers for GHP systems: horizontal underground heat exchangers, and deep well heat exchangers (Figure 1).




a) **Horizontal (slinky) GHEs** b) **Vertikal (single U-pipe) GHEs**

c) **Pile (spiral) GHEs** d) **Deep borehole (coaxial) GHEs**

Figure 1. Basic schemes of four types of underground heat exchangers.

Horizontal underground heat exchangers are not widely used in the world because they usually require a large area. Therefore, we will not dwell on horizontal underground heat exchange devices.

The main advantages and disadvantages of four types of underground heat exchangers are presented in Table 1.

A summary of the advantages and disadvantages of four typical GHEs.

| Form | | Advantages | Disadvantages | Applicable situations |
|---|---|---|---|---|
| Horizontal GHE | U-shaped pipe, serpentine pipe and Slinky, etc | Provide heating and cooling; lower construction cost of ditching compared to drilling cost of VBGHEs. | Heat transfer efficiency is affected by external climate; high pipe consumption; large occupied land area. | Suitable for single-family homes and some occasions requiring or suitable for large excavation. |
| VBGHE | Single U-pipe or double U-pipes | Provide heating and cooling; stable working performance; less occupied land area. | Drilling cost is affected by geological conditions; heat transfer performance is affected by borehole spacing and annual cooling and heating load. | Suitable for the buildings with well-balanced heating and cooling loads. |
| PGHE | W-shaped pipe, single or multiple | Provide heating and cooling; lower construction cost and smaller land area; higher heat transfer rate per pile meter compared to VBGHEs. | After long-term operation, temperature change may affect the mechanical properties of pile foundation. | Suitable for projects where land is in short supply or drilling is not suitable. |
| DBGHE | U-pipes and spiral pipe, etc coaxial pipes or U-bend pipe | Stable heat source; larger heat extraction capacity compared to VBGHEs; higher energy efficiency and smaller occupied land area for drilling. | Only for space heating, cannot provide the cooling; the drilling is difficult and the initial investment is high. | Suitable for heating-dominated buildings in cold regions. |

The first information about GHPs is a patent obtained in 1912 [3]. The period of rapid development of GHPs in North America and Europe corresponds to the 1970 s after the first oil crisis. Most of the numerous studies have focused on heat transfer modeling and design of GHPs.

## II. MATERIALS AND METHODS

The current state of scientific research on vertical geothermal heat exchangers, piled geothermal heat exchangers and deep-well geothermal heat exchangers and referenced in the Science Direct database is shown in Figure 2.

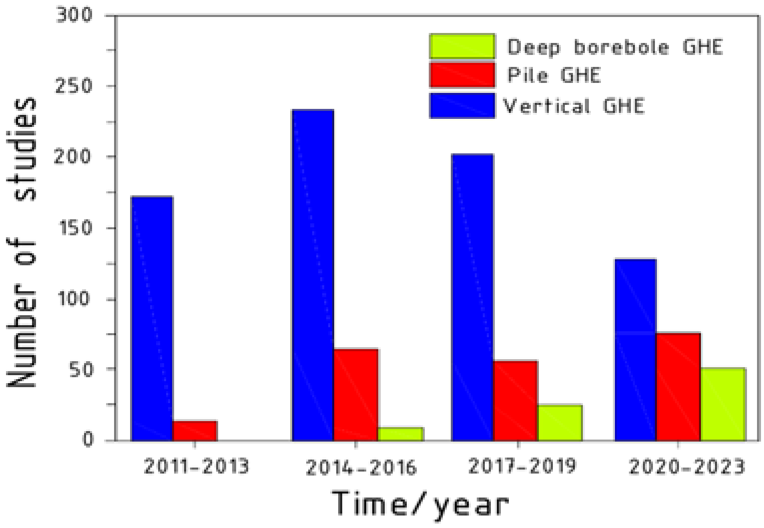



Fig. 2. Statistical diagram of relevant literatures published in past decade.

As can be seen from Figure 2, the number of research works on deep-well underground heat exchange devices was the largest in 2016, and then decreased. Compared to 2016, the number of articles published in 2021 decreased by 40%. On the other hand, studies on piled ground heat exchangers and deep well ground heat exchangers are still popular. These analyzes can be divided into three main categories. Analyzes of the first category are devoted to the modeling and design of deep-well underground heat exchangers and thermal testing of piled underground heat exchangers [3-7]. Analyzes of the second category are related to hybrid systems with GHPs that accumulate energy [8,9] and GHPs with solar energy [10,11]. Analyzes of the third category are related to research works [12,13] related to underground heat exchange devices with deep wells. The level of modeling of the heat transfer between the underground heat exchanger and the ground is also similar, and the modeling experience can be compared with each other. Also, knowledge of different forms of GHP is very useful for engineers.

Analytical modeling Most analytical methods of deep-well underground heat exchangers are based on either a linear heat source [14,43] or a cylindrical heat source. The first analytical solution of an infinite linear source of single-well thermal conductivity was proposed by Ingersoll and Plasslar [15], which served as the basis for the development of the subsequent design program. In 1986, Eskilson [16] introduced the principle of superposition to the thermal analysis of a well system. Using the principle of superposition based on temperature fluctuations in one well with constant heat flow, it was possible to obtain real temperature fluctuations in several wells under conditions of variable heating and cooling loads. This method of superposition is widely used in engineering projects of small vertical underground heat exchangers, due to the comprehensibility of the physical nature of the method and ease of calculation [6-17]. Models with both a linear and cylindrical heat source allow determining the homogeneous heat flow on the well wall and the homogeneous thermal property of the soil in deep wells, which leads to deviations from the real state.

Abdelaziz et al proposed a new multilayer FLS model to calculate the temperature response within different layers based on the finite line source model and the principle of superposition. In the multi-layer model, the GHP is divided into several segments, and the reaction of the soil temperature at a given point can be determined by summing up the individual contributions of all these segments. The estimation results of the multilayer model are in good agreement with the results obtained using the finite element method. The temperatures measured by the multilayer model are compared with the temperatures measured by the finite element model for homogeneous soil. According to the results, the incorrect estimation of the temperature measurement ranges from 10 to 25%, which is due to the fact that the soil is assumed to be homogeneous [18,44]. Pan et al. [19] proposed a multilayer model with a cylindrical heat source, which is similar to the multilayer FSL model. This model is designed for vertical underground heat exchangers in layered soil and allows analytical expression of the temperature reaction of the soil using the integral transformation method. According to the results, due to the difference in thermal properties between the soil layers, additional vertical heat transmission was observed through the boundaries of the layers. The above analytical models do not take into account the thermoconvective effect of groundwater flow on underground heat exchangers. In fact, the presence of groundwater has a significant effect on the thermal characteristics of underground heat exchangers [20]. Diao et al. [21] proposed a moving infinite linear source model to calculate the convective effects of groundwater and obtained an analytical solution of the soil temperature response in an infinite homogeneous pore medium with uniformly changing water.

Sutton et al. [22] developed a soil resistivity estimation model that takes groundwater convection into account based on a moving line source solution. The combined value differs from the grounding value only in terms of conductivity. Compared with the traditional model of a linear source, the solution shows that the flow of groundwater has a significant effect on the heat transfer process. Later, Molina-Jiraldo et al. [23] proposed an infinite linear moving source model to take into account the heat transfer and groundwater flow. This model is mainly useful for long-term modeling and overcomes the limitations of the infinite linear moving source model. Underground heat exchange devices can be placed in different layers according to the depth of wells with different hydrogeological and thermal characteristics. Thus, when the difference in groundwater flow between the layers is large, it is necessary to take into account the inhomogeneity of the soil in order to evaluate the long-term effectiveness of GHP systems. For vertical underground heat exchangers, improved models were developed that take into account the influence of groundwater in multi-layer geologies [24,25]. The results show that the low velocity of groundwater weakens the thermal interaction between adjacent layers. In the case of low groundwater flow rate, soil homogeneity is achieved between the layers. Conversely, in transitional conditions, especially at layer boundaries, a multi-layer approach is appropriate. According to the groundwater flow MFLS model, the inverse calculation methodology was studied to determine the direction and speed of the groundwater flow passing through the underground heat exchange device with the verified temperature fluctuation at several points [26]. that is, it provides an accurate value of the velocity of groundwater, then it is possible to analyze the efficiency of heat transfer in underground heat exchangers using the estimated parameters of groundwater. Pan et al. [27] developed a new analytical model with three different upper boundary conditions, which can be used to study the temperature fluctuation in a deep-well underground heat exchanger. According to the results, the average temperature calculated in the deep-well underground heat exchanger when using Dirichlet boundary conditions is 14.1 and 8.5% lower than when using Neumann and Robin boundary conditions.

Numerical modeling Numerical methods of researching heat transfer in underground heat exchangers are carried out using the finite element method or the finite difference method based on continuous numerical schemes. With the development of computer technology, numerical calculations have become the main tool in the study of heat transfer and are the main tool in the theoretical study of underground heat exchangers. This direction takes into account the very specific case, i.e., soil inhomogeneity, and provides research based on boundary conditions without accepting reduced conditions. The first three-dimensional numerical model was proposed by Hellström [28], and it consists of three parts: heat exchange inside the well, local heat exchange of individual wells, and a global model connecting individual wells. The model was then developed in TRNSYS software [29]. In recent years, a number of models with various simplified conditions have been developed, such as the temperature gradient model of groundwater flows [30] and the model of different layers of rock and soil [31,45].

A globe numerical model has been developed, which provides a reduction in calculation time in the case of parallel connection of underground heat exchange devices while maintaining accuracy. This model is a combination of 1D model of heat exchange inside the well and 3D models of heat exchange outside the well [32]. Based on the CFD model, a numerical study of coaxial small deep well underground heat exchangers was carried out, and in this study, the effect of several structural parameters, inlet flow, inlet fluid temperature, inner pipe material and outer pipe diameter on the heat transfer characteristics was studied [33]. Thus, the use of numerical methods for the design of underground heat exchangers for engineering purposes is very inconvenient due to the strong differentiation of length and time scales, and numerical methods require a long calculation time. G-function and short time step model. Eskilson and Claesson [34-36] introduced a new concept of g-function to simplify the calculation of soil temperature response. This function is defined as the dimensionless temperature coefficient of vibration. It is related to the temperature of the well wall and the actual heat coefficient of the well to the depth of the ground. This hybrid model is a combination of analytical and numerical solutions. A two-dimensional numerical calculation was carried out using unsteady finite difference equations in the radial-axial coordinate system for a single well in homogeneous soil with constant initial and boundary conditions. The solution was obtained using a basic step heat pulse. The response of the well field to a single pulse was then expressed using the g-function. Later, Yavuztürk and Spitler [37] presented a short-time step model for modeling unsteady heat transfer in deep-well subsurface heat exchangers with an accuracy of hours or less. Numerical results are expressed by the coefficient of variation at a short time step. Prieto and Chimminos [38] developed the equivalent well method, which allows calculation of the temperature fluctuation function for more than a thousand well deposits in a few seconds. Later, a semi-analytical method for calculating the g-function of the well area with mixed wells connected in series and parallel was presented [39]. As mentioned above, the most widely used methodology for modeling deep-well underground heat exchange devices is to divide the heat exchange zone in the well into two parts: a stable heat exchange part inside the well and an unstable heat exchange part outside the well. It can be considered stable due to its small size and heat capacity compared to soil. Researches in the last decade have been devoted to the study of the short-time response of the underground heat exchanger, they are aimed at determining the optimal design of hybrid GHPs, testing their vibration in places, hourly modeling and optimal control of work [40]. Using the electrical analogy approach, an improved "power resistance model" was developed to calculate the heat capacity of the well [41]. A new model with moderate power and thermal resistance is used to solve the problem of heat transfer in an unstable state. To study the heat processes in the underground heat exchanger, Li et al. proposed a linear source model with a composite medium [42]. This model takes into account not only the effect of solution heat capacity, but also the difference between soil and solution, and also allows modeling of different configurations of U-shaped pipes.

## SUMMARY

In this analytical article, the latest achievements and researches on the modeling of underground heat exchangers with vertical wells were analyzed. For the in-depth study of heat transfer processes in a deep-well underground heat exchanger, it was justified that it is not used in engineering projects in comparison with analytical models with different degrees of accuracy, and instead, it was justified to use the function g.

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